

## Investigation of Phase Transition in the One-Dimensional Phospholipids Model by Combinatorial Factor Method

S. Ranjbar\* and S. Taslimian

Chemistry College, Razi University, 67149-67346, Kermanshah, Iran

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In this study, using a one-dimensional phospholipids model, we have introduced an exact combinatorial factor method for the investigation of order-disorder in the phospholipids lattice. Our assumption was that the lattice was composed of six groups, and for simplicity, we assumed that the total energy of the lattice can be considered as the rotational energy of gauche molecules and the nearest neighbor interactions between *trans* molecules. Using the combinatorial factor method (CFM), the total energy and the corresponding constraints, the Helmholtz free energy was minimized. Finally, the thermodynamic properties of the one-dimensional lattice including the internal energy, entropy, and heat capacity were determined, exactly. The results show that, adopting such a model and under specific conditions, a phase transition, similar to the one in Onsager transition, takes place.

**Keywords:** Phase transition, Combinatorial factor method, Phospholipids model, Helmholtz free energy, Order-disorder

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### INTRODUCTION

Phospholipids are abundant in all biological membranes. Phospholipids derived from glycerol are called phosphoglycerides. A phosphoglyceride consists of a glycerol backbone, two fatty acid chains, and a phosphorylated alcohol [1]. In these compounds, about half of the fatty acyl groups are saturated, that is, they contain no double bonds. The other half of the fatty acyl groups contain one or more double bonds and are therefore derived from unsaturated or polyunsaturated fatty acids [2]. The configuration of double bonds in unsaturated fatty acids is nearly always *cis*. The length and the degree of unsaturation of fatty acids chains in membrane lipids have a profound effect on membrane fluidity.

The two important kinds of conformations of phospholipids chains are the ordered and disordered states. Experimental observations show that the ordered and

disordered states could be converted to each other through a gel-to-liquid-crystalline transition. The fatty acyl chains in bilayer membranes exist in an ordered rigid state or in a relatively disordered fluid state. In the ordered state, all the C-C bonds have *trans* conformation, whereas in the disordered state, some are in the gauche conformation. The transition from the rigid (all *trans*) to the fluid (partly gauche) state occurs rather abruptly as the temperature is raised to above  $T_m$ , the melting temperature [1]. The Differential Scanning Calorimetry (DSC) studies show that the phase transition from solid to liquid state takes place over an infinitely narrow temperature range.

Phospholipid monolayers at an oil-water interface are treated as two-dimensional regular solutions made up of three components, namely, singly dispersed phospholipid molecules, clusters of phospholipid molecules, and empty sites occupied by water and oil molecules. A simple surface equation of state is derived and used to explain the phase transition that occurs in the monolayer. The equation contains

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\*Corresponding author. E-mail: shm.ranjbar@gmail.com

two terms: one is due to the two-dimensional mixing entropy and the other one arises from the intermolecular interactions calculated in the nearest-neighbor approximation. The equation reduces to the two-dimensional van der Waals equation of state when no clustering occurs [3].

The theory of nonequilibrium phase transition is an extremely active area in theoretical physics. It is applied in a wide range of natural and social sciences. The nonequilibrium phase transition is usually considered as a stochastic process. The respective stochastic equations are solved either by numerical integration or different approximations applied in order to simplify the mathematical problem, for example, the adiabatic elimination techniques and the method of partial distribution functions. The processes are directly simulated by the Monte Carlo method [4].

For the description of the lipid bilayer gel-fluid phase transition a number of statistical and mechanical models have been proposed. These models include molecular mean-field models [5-11], Monte Carlo simulations [12-14], and molecular dynamics simulations [15,16]. Also, using Ising lattice model, this transition has been studied theoretically from two-state [17,18] to the 10-state Pink model [12,19,20].

In this work, based on the combinatorial factor method [21,22] and a simple one-dimensional model, the thermodynamic properties of the phospholipids chains are investigated.

## RESULTS AND DISCUSSION

Consider a one-dimensional phospholipid chains which are placed in an inert solvent. Let us assume that the solvent molecules and phospholipid chains can be considered as one-dimensional lattice of  $N$  units. Irrespective of the solvent molecules, each of these units can be labeled as rigid (all *trans*) or liquid (partly *gauche*). At low temperatures (near the absolute zero temperature), the lattice is in the ordered state so that all of the units are in rigid form. When the temperature increases, the lattice becomes disordered and the liquid units appear. Suppose that the units may be classified into three different types;  $t$  (*trans* molecules),  $g$  (*gauche* molecules), and  $h$  (solvent molecules) with numbers  $N_t$ ,  $N_g$  and  $N_h$ , respectively ( $N_p \rightarrow \infty$ ,  $p = t, g, h$ ), so that

$$N = N_t + N_g + N_h$$

where  $N$  is the number of units or pairs (the nearest neighbor pairs). These pairs can be classified into nine different types as  $tt, tg, th, gt, gg, gh, ht, hg$ , and  $hh$  so that their number may be indicated as  $N_{tt}, N_{tg}, N_{th}, N_{gt}, N_{gg}, N_{gh}, N_{ht}, N_{hg}$ , and  $N_{hh}$ , respectively.

Let us consider a regular one-dimensional configuration for a lattice as

...ttttt...  
 ttgtgtg...gtgththth...hthtgggggg...gggghghgh...hghghhhhh  
 ...

According to the above classification, we can impose the following constraints:

$$N_{tt} + N_{tg} + N_{th} = N_t, \tag{1}$$

$$N_{gt} + N_{gg} + N_{gh} = N_g, \tag{2}$$

$$N_{ht} + N_{hg} + N_{hh} = N_h, \tag{3}$$

If the mole fraction of phospholipids be  $\theta$ , then, we have

$$N_{tt} + N_{tg} + N_{th} + N_{gt} + N_{gg} + N_{gh} = N\theta, \tag{4}$$

$$N_{ht} + N_{hg} + N_{hh} = N(1 - \theta), \tag{5}$$

where

$$N_{pq} = N_{qp}, \quad p, q = t, g, h \tag{6}$$

Since the interaction energies between phospholipid chains are very complicated, we may assume that the total energy of the lattice is the sum of the nearest neighbor interactions between trans-trans units with the coupling constant  $-U$  ( $U > 0$ ) and the rotational energy of *gauche* units with the rotational constant  $J$  ( $J > 0$ ) For the sake of simplicity, we assume that the multiplicity of each rotational energy level is the unit which may be labeled as  $r = 0, 1, 2, \dots, r_{\max}$ , where  $r_{\max}$  is the maximum rotational energy level of the disordered or *gauche* units. Thus, according to the above assumptions, the energy of each disordered unit which be positioned at  $r$ th level, is  $rJ$ .

Using these assumptions, the total energy of the model can be written as

$$E = -UN_{tt} + J \sum_{r=0}^{r_{\max}} rM_r, \tag{7}$$

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where  $M_r$  is the number of the disordered or gauche units which lie in the  $r$ th rotational energy level, which attentive to it, the following constraint must be imposed too,

$$N_g = \sum_{r=0}^{r_{\max}} M_r. \quad (8)$$

Irrespective of distribution of disordered units in rotational level, the model may be considered as one-dimensional simple model with the nearest neighbor interactions. For this model, the combinatorial factor,  $\Omega_1$ , is [21]

$$\Omega_1 = \frac{N_t!}{N_{tt}!N_{tg}!N_{th}!} \frac{N_g!}{N_{gt}!N_{gg}!N_{gh}!} \frac{N_h!}{N_{ht}!N_{hg}!N_{hh}!} \quad (9)$$

Now in the  $r$ th rotational level, there are  $M_r$  indistinguishable disordered units, therefore we must consider following distribution too,

$$\Omega_2 = \frac{N_g!}{\prod_{r=0}^{r_{\max}} M_r!}. \quad (10)$$

Because, these distributions ( $\Omega_1, \Omega_2$ ) are independent from each other, the exact total combinatorial factor,  $\Omega$ , for such model is

$$\Omega = \Omega_1 \Omega_2 = \prod_p \prod_q \prod_{r=0}^{r_{\max}} \frac{N_p! N_g!}{N_{pq}! M_r!} \quad (11)$$

Applying Stirling's approximation, the reduced Helmholtz free energy,  $A/NkT = E/NkT - \ln\Omega/Nk$  can be written as

$$\frac{A}{NkT} = -a j n_{tt} + j \sum_{r=0}^{r_{\max}} r m_r + \sum_p \sum_q n_{pq} \ln n_{pq} + \sum_{r=0}^{r_{\max}} m_r \ln m_r - \sum_p n_p \ln n_p - n_g \ln n_g, \quad (12)$$

where  $a \equiv U/J$  and also,

$$j = \frac{J}{kT}, \quad n_p = \frac{N_p}{N}, \quad n_{pq} = \frac{N_{pq}}{N}, \quad m_r = \frac{M_r}{N}. \quad (13)$$

To obtain the equilibrium state, the Helmholtz free energy must be minimized. Defining

$$x \equiv \sqrt{n_{tt}/n_{gg}}, \quad y \equiv \sqrt{n_{th}/n_{tt}}, \quad \lambda \equiv e^{-j}, \quad (14)$$

$$w = \sum_{r=0}^{r_{\max}} \lambda^r = \frac{1-\lambda^{1+r_{\max}}}{1-\lambda},$$

and using Eqs. (1) to (6), (8), (13) and also applying the Lagrange multipliers method, we have

$$m_r = m_0 \lambda^r \quad r = 0, 1, 2, \dots, r_{\max}, \quad (15)$$

$$n_t = \frac{\lambda^a}{m_0} x^2 n_g^2 \quad (16)$$

$$n_t = \frac{[1+\lambda^{a/2}(y+1/x)](1-\theta)}{(\lambda^{a/2}+y+1/x)y} \quad (17)$$

$$n_t = \theta \frac{1-\theta}{xy} \quad (18)$$

$$m_0 = \frac{(1-\theta)}{xyw} \quad (19)$$

$$n_{tt} = \frac{(1-\theta)}{(\lambda^{a/2}+y+1/x)y} \quad (20)$$

$$n_g = \frac{(1-\theta)}{xy} \quad (21)$$

$$n_{tg} = \frac{\lambda^{a/2}}{x} n_{tt} \quad (22)$$

$$n_{th} = y n_{tt} \lambda^{a/2} \quad (23)$$

$$n_{gh} = \frac{y}{x} n_{tt} \quad (24)$$

where  $x, y$  can be determined from following equations

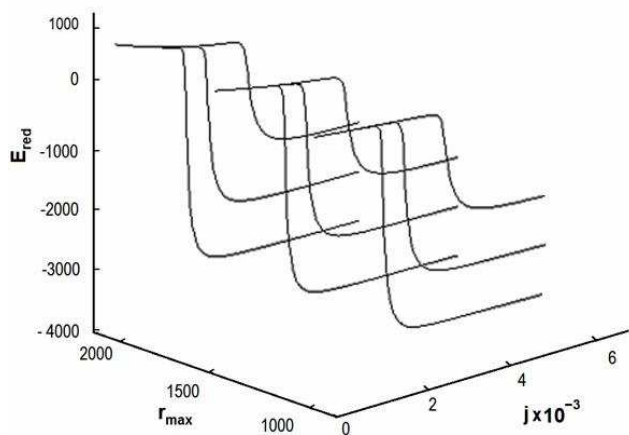
$$w^2 \lambda^a (1-\theta) x^3 + w \lambda^{a/2} (2\theta-1) x^2 + (w \lambda^{a/2} - \theta \lambda^{-a/2}) x - 1 = 0, \quad (25)$$

$$y = \frac{1-\theta}{\theta x} (x^2 w \lambda^a + 1) \quad (26)$$

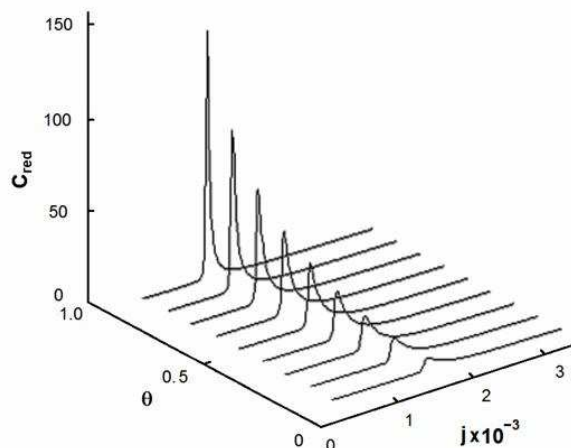
Using the above results, Eq. (12) may be simplified as

$$\frac{A}{NkT} = -a j \theta + \theta \ln \frac{n_{tt}}{n_t} + (1-\theta) \ln \frac{n_{hh}}{1-\theta}. \quad (27)$$

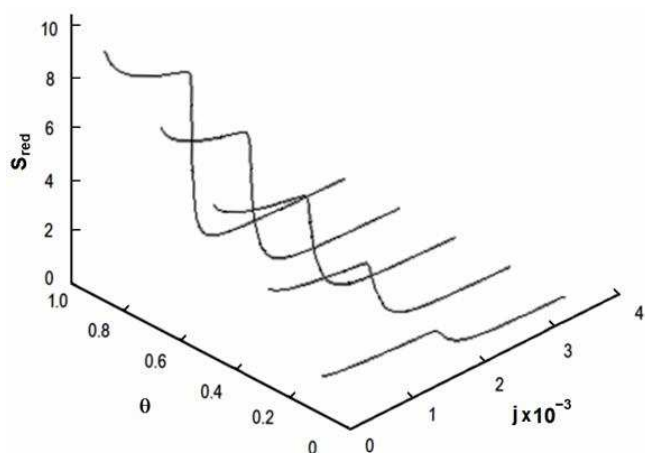
Now, through solving the nonlinear Eq. (25), we can find  $n_{pq}, n_t$  by using which, and applying Eq. (27), we can derive thermodynamic properties of the lattice. The results of such calculations for the internal energy, entropy, and heat capacity



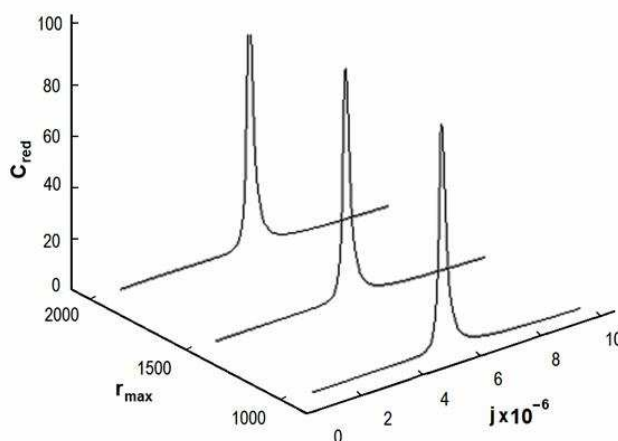
**Fig. 1.** The reduced internal energy,  $E_{red}$  ( $-E/Nk$ ), vs.  $r_{max}$  and  $j$  for  $\theta = 0.8$  and  $a = 2000, 3000$  and  $4000$  (from up to down ).



**Fig. 3.** The same as Fig. 2 for the reduced heat capacity,  $C_{red}$  ( $C/Nk$ ).



**Fig. 2.** The reduced entropy,  $S_{red}$  ( $S/Nk$ ), vs.  $\theta$  and  $j$  for  $\alpha = 5000$  and  $r_{max} = 10^6$ .



**Fig. 4.** The reduced heat capacity,  $C_{red}$ , vs.  $r_{max}$  and  $j$  for  $\theta = 0.95$  and  $\alpha = 1000$ .

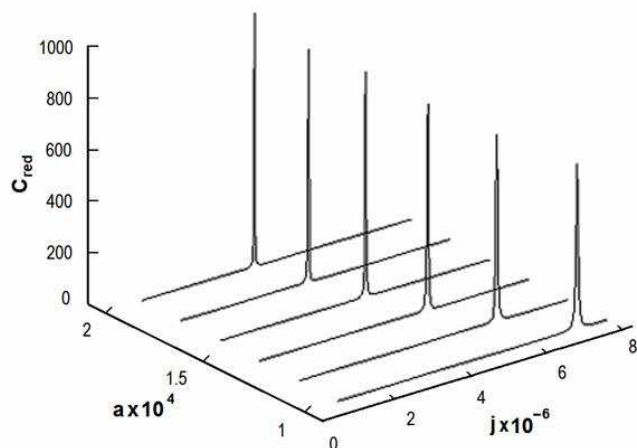
are given in Figs. 1 to 6.

## CONCLUSIONS

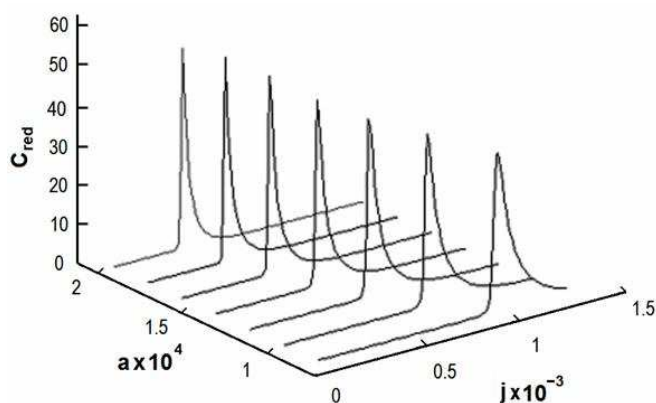
In this work, we used the combinatorial factor method to investigate the phase transition in phospholipid chains. In this way, using a one-dimensional model and introducing an exact combinatorial factor, the Helmholtz free energy was minimized. The results are given in Figs. 1 to 6. In these

figures, there are some important points which can be expressed as follows:

1. We see that, all of the thermodynamic properties are dependent on two parameters  $a$  and  $r$ .
2. At low temperatures ( $j \rightarrow \infty$ ), the ordered form is stable whereas, at high temperatures ( $j \rightarrow 0$ ), the disordered form is stable (Fig. 2).
3. In Figs. 5 and 6, we see that, at specific temperatures and high values of  $r_{max}$  and  $a$ , the heat capacities show an



**Fig. 5.** The reduced heat capacity,  $C_{red}$ , vs.  $a$  and  $j$  for  $\theta = 1$  and  $r_{max} = 10^6$ .



**Fig. 6.** The same as Fig. 5 for  $\theta = 0.5$ .

Onsager phase transition, while in Figs. 3 and 4 at small values of  $r_{max}$  and  $a$ , the heat capacities show only a diffuse or continuous transition [25]. Experimental results such as DSC show that, at low temperatures and over an infinitely narrow temperature range, the heat capacity shows a phase transition (like Onsager transition) [26], which is comparable to our results (Fig. 5).

Finally, it is obvious that the approach in question in this work can be used for two-dimensional phospholipids chains and for different types of motions and interactions of phospholipid molecules. This model is very similar to the

actual lattice, but we must admit that, for these conditions, the calculation of combinatorial factor is rather difficult which we will take up in future.

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## REFERENCES

- [1] L. Stryer, *Biochemistry*, 4<sup>th</sup> ed., W.H. Freeman and Company, New York, 1995.
- [2] J.D. Rawn, *Biochemistry*, Neil Patterson Publishers, Burlington, North Carolina, USA, 1989.
- [3] K.S. Birdi, *Lipid and Biopolymer Monolayers at Liquid Interfaces*, Plenum, New York, 1989.
- [4] I.P. Sugar, *J. Phys. Chem.* 93 (1989) 5216.
- [5] S. Marcelja, *Biochim. Biophys. Acta* 367 (1974) 165.
- [6] J.P. Meraldi, J. Schlitter, *J. Biochim. Biophys. Acta* 645 (1981) 183.
- [7] J.P. Meraldi, J. Schlitter, *J. Biochim. Biophys. Acta* 645 (1981) 193.
- [8] D.W.R. Gruen, *Biochim. Biophys. Acta* 595 (1981) 161.
- [9] D.W.R. Gruen, *J. Phys. Chem.* 89 (1985) 146.
- [10] A. Ben-Shaul, I. Szleifer, W.M. Gelbart, *J. Chem. Phys.* 83 (1985) 3597.
- [11] D.R. Fattal, A. Ben-Shaul, *Biophys. J.* 67 (1994) 983.
- [12] O.G. Mouritsen, A. Boothroyd, R. Harris, N. Jan, T. Lookman, L. MacDonald, D.A. Pink, M.J. Zuckerman, *J. Chem. Phys.* 79 (1983) 2027.
- [13] O.G. Mouritsen, J.H. Ipsen, K. Jørgensen, M.M. Sperotto, Z. Zhang, E. Corvera, D.P. Fraser, M.J. Zuckermann, in: G.F. Luckhurst (Ed.), *Computer Simulation of Phase Transitions in Nature's Preferred Liquid Crystal: The Lipid Bilayer Membrane*. In *Computer Simulation of Liquid Crystals*. Kluwer Academic Publishers, Dordrecht, The Netherlands, 1992.
- [14] I.P. Sugar, T.E. Thompson, R.L. Biltonen, *Biophys. J.* 76 (1999) 2099.
- [15] P. van der Ploeg, H.J. Berendsen, *J. Chem. Phys.*

- 76 (1982) 3271.
- [16] K. Tu, D.J. Tobias, M.L. Klein, *Biophys. J.* 69 (1995) 2558.
- [17] I.P. Sugar, R.L. Biltonen, N. Mitchard, *Methods Enzymol.* 240 (1994) 569.
- [18] V.P. Ivanova, T. Heimburg, *Phys. Rev. E* 63 (2001) 1914.
- [19] D.A. Pink, T.J. Green, D. Chapman, *Biochemistry* 19 (1980) 349.
- [20] O.G. Mouritsen, in: R. Brasseur (Ed.), *Computer Simulation of Cooperative Phenomena in Lipid Membranes*. In *Molecular Description of Biological Membrane Components by Computer-Aided Conformational Analysis*. CRC Press, Boca Raton, 1990.
- [21] S. Ranjbar, *Chem. Phys.* 326 (2006) 483.
- [22] S. Ranjbar, *J. Iran. Chem. Soc.* 7 (2010) 84.
- [23] D.A. Mcquarri, *Statistical Thermodynamics*, Harper and Row, New York, 1973.
- [24] L.E. Reichal, *A Modern Course in Statistical Physics*, John Wiley, New York, 1998.
- [25] G. Wang, S. Li, H. Lin, E.E. Brumbaugh, C. Huang, *J. Biol. Chem.* 274 (1999) 12289.